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Mathematics: analysis and approaches

Higher level

Paper 3

20 May 2026

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 15 minutes

Instructions to students

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 30]

This question considers different methods of summing powers of integers.

The sum of the first n positive integers, each raised to the power zero, is given by

$$S_0(n) = 1^0 + 2^0 + \dots + n^0 \text{ where } n \in \mathbb{Z}^+.$$

(a) (i) Find the value of $S_0(4)$. [1]

(ii) Write down a simplified expression for $S_0(n)$. [1]

The sum of the first n positive integers, each raised to the power 1, is given by

$$S_1(n) = 1^1 + 2^1 + \dots + n^1 \text{ where } n \in \mathbb{Z}^+.$$

(b) Use the fact that $S_1(n)$ is the sum of n terms of an arithmetic sequence to show

that $S_1(n) = \frac{n^2}{2} + \frac{n}{2}$. [3]

(This question continues on the following page)

(Question 1 continued)

The sum of the first n square numbers is given by

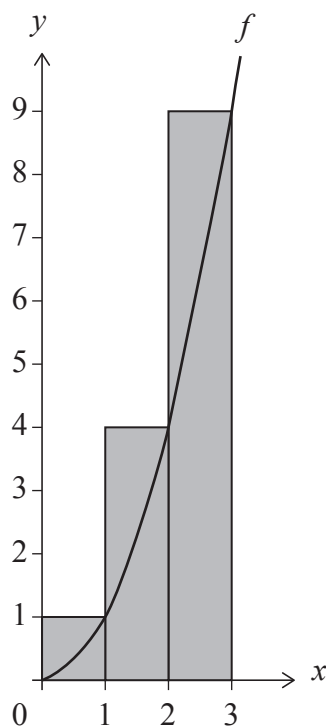
$$S_2(n) = 1^2 + 2^2 + 3^2 \dots + n^2 \text{ where } n \in \mathbb{Z}^+.$$

(c) Find $S_2(5)$. [1]

(d) The following graph shows three rectangles, where the heights of the rectangles are consecutive square numbers, and the width of each rectangle is 1.

The graph may be extended in a similar way to include any number of rectangles.

The curve $y = f(x)$ passes through the top right corner of each rectangle.



(i) Write down an expression for $f(x)$. [1]

(ii) By considering the total area of the first n rectangles and the area under the curve, show that $S_2(n) > \frac{n^3}{3}$. [4]

(iii) Use a similar method to show that $S_2(n) < \frac{n^3}{3} + n^2 + n$. [4]

(This question continues on the following page)

(Question 1 continued)

Consider the sequence $u_n = n^3 - (n-1)^3$ for $n \in \mathbb{Z}^+$. Some of the values of u_n and $\sum_{r=1}^n u_r$ are shown in the following table.

n	u_n	$\sum_{r=1}^n u_r$
1	1	1
2	7	8
3	a	b
4	37	64

- (e) Find the value of
- (i) a ; [1]
 - (ii) b . [1]
- (f) (i) Suggest an expression for $\sum_{r=1}^n u_r$ in terms of n . [1]
- (ii) Use the definition of u_n to prove that your suggestion is true. [3]
- (g) (i) Find a quadratic expression for u_n . [2]
- (ii) Hence, write $\sum_{r=1}^n u_r$ in terms of $S_2(n)$, $S_1(n)$ and $S_0(n)$. [2]
- (iii) Hence, show that $S_2(n) = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$. [3]
- (iv) Show that the expression for $S_2(n)$ given in (g)(iii) is consistent with the inequalities in part (d). [2]

2. [Maximum mark: 25]

This question asks you to explore curves in the complex plane defined by $r = a + b \sin \theta$, where a and b are real constants.

A complex number z can be represented by a point in the complex plane whose position is determined by the modulus, r , and the argument, θ , of z .

The set of points which satisfy a given relationship between r and θ form a curve in the complex plane.

Consider a family of curves defined by $r = a + b \sin \theta$, where a and b are real constants.

First consider the case where $a = 4$ and $b = 0$ so that $r = 4$.

The curve defined by $r = 4$ is the set of points in the complex plane which represent complex numbers with a modulus of 4.

(a) Suppose that z is a complex number such that $r = 4$.

Find all possible values of z such that

(i) z is purely real; [1]

(ii) z is purely imaginary; [1]

(iii) the argument of z is $\frac{\pi}{4}$. [1]

(b) Sketch the curve defined by $r = 4$ on an Argand diagram, showing clearly the values of the intercepts with the real and imaginary axes. [2]

Now consider the case where $a = 0$ and $b = 4$ so that $r = 4 \sin \theta$, where $0 \leq \theta \leq \pi$.

(c) Suppose that z is a complex number such that $r = 4 \sin \theta$, where $0 \leq \theta \leq \pi$.

(i) Write down the greatest possible value of r . [1]

(ii) For this value of r , find z . [2]

(d) Use the identities $x \equiv r \cos \theta$ and $y \equiv r \sin \theta$ to show that the points on the curve defined by $r = 4 \sin \theta$ lie on the curve $x^2 + (y - 2)^2 = 4$, where $x = \operatorname{Re}(z)$ and $y = \operatorname{Im}(z)$. [4]

(This question continues on the following page)

(Question 2 continued)

(e) Consider the curve $x^2 + (y - 2)^2 = 4$.

(i) Find an expression for $\frac{dy}{dx}$ in terms of x and y . [3]

(ii) Hence or otherwise, find the values of x for which the gradient of the curve is undefined. [2]

(f) The curve defined by $r = 4 \sin \theta$, where $0 \leq \theta \leq \pi$, is a circle.

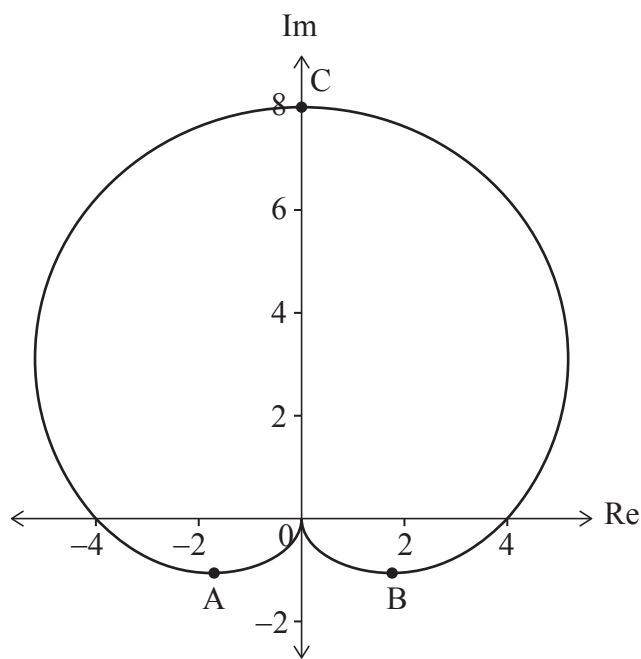
Sketch this curve on an Argand diagram, showing clearly the values of any intercepts with the real and imaginary axes. [1]

(This question continues on the following page)

(Question 2 continued)

Finally, consider the case where $a = 4$ and $b = 4$.

The curve defined by $r = 4 + 4 \sin \theta$, where $0 \leq \theta \leq 2\pi$, is shown on the following Argand diagram. The curve has a horizontal tangent at point A, point B and point C. The real parts of the numbers represented by A, B and C are negative, positive and zero, respectively.



- (g) Use the identities $x \equiv r \cos \theta$ and $y \equiv r \sin \theta$ to show that the gradient of the curve at the point $(r \cos \theta, r \sin \theta)$ is given by $\frac{dy}{dx} = \frac{\cos \theta (1 + 2 \sin \theta)}{1 - \sin \theta - 2 \sin^2 \theta}$. [5]
- (h) Hence, find the value of θ which corresponds to the point A. [2]
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